4401. The relationship $x^4+y^4=1$ defines a closed curve.

(a) Verify that, in the positive quadrant, the curve may be written $x=\sqrt{\cos t}$, $y=\sqrt{\sin t}$.

(b) Explain why $\sqrt{\cos t}\geq \cos t$ always holds, if the square root is well defined.

(c) Hence, sketch $x^4+y^4=1$ and the unit circle on the same set of axes.

4402. The process of *completing the cube* is similar to that of completing the square.

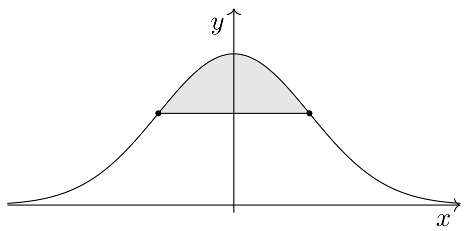
(a) Show that ax^3+bx^2+cx+d can be written in the form $p(x+\alpha)^3+q(x+\alpha)+r$.

(b) Hence, prove that every cubic has rotational symmetry.

4403. The bell curve of the standard normal distribution $X\sim N(0,1)$ is defined by the function

$$\phi(x)=\tfrac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}.$$

A chord is drawn to the curve $y=\phi(x)$ between its two points of inflection:



Find, to 3sf, the area of the shaded region.

4404. Show that $\displaystyle\int_5^8 \frac{2x^2}{x^2-16}\,dx=6+4\ln 3.$

4405. Let P be a polynomial curve $y=\mathrm{f}(x)$ of degree $n\geq 2$, with no points of inflection. This question is a proof that no three points on P are collinear.

Assume, for a contradiction, that curve P has at least three collinear points. Let the first three be $(a,\mathrm{f}(a))$, $(b,\mathrm{f}(b))$ and $(c,\mathrm{f}(c))$, where $a<b<c$. Let the line through these points be $y=mx+c$. Let Q be the curve $y=\mathrm{h}(x)$, where

$$\mathrm{h}(x)=\mathrm{f}(x)-mx-c.$$

(a) Explain why, without loss of generality, it may be assumed that $\mathrm{f}''(x)\geq 0$ for all x.

(b) Hence, show that $\mathrm{h}''(x)\geq 0$ for all x.

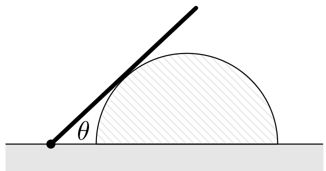
(c) Describe the x intercepts of Q.

(d) Show that $\mathrm{h}(x)<0$ for all $x\in(a,b)\cup(b,c)$.

(e) Hence, complete the proof.

4406. Either prove or disprove the following statement: "If events X and Y are dependent, and events Y and Z are dependent, then events X and Z are dependent."

4407. A smooth, uniform rod of mass m is freely hinged to a flat, horizontal surface. It leans up against a half-cylindrical block of mass m, which is at rest on the surface. The point of contact between rod and half-cylinder is at the centre of mass of the rod. The system is in equilibrium.



Show that the coefficient of friction between the half-cylinder and the horizontal surface satisfies

$$\mu\geq\frac{\sin 2\theta}{\cos 2\theta+3}.$$

4408. The following graphs are tangent to one another:

$$x^2y^3-2=xy^{\frac{3}{2}},$$
$$3y=2x+5.$$

Show that they intersect at exactly one point other than their point of tangency.

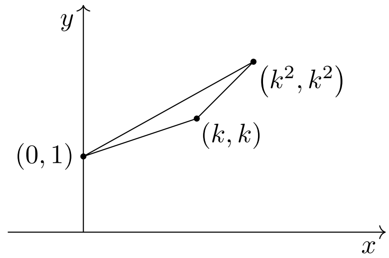
4409. Four distinct vertices are chosen at random from those of a regular 16-gon. Find the probability that the vertices form a square.

4410. Simultaneous equations are given as

$$P\sin 2t=1,$$
$$P\tan t=2.$$

Solve these for $P\in\mathbb{R}$ and $t\in[0,2\pi)$, giving your answers exactly.

4411. A triangle has vertices at $(0,1)$, (k,k) and $\left(k^2,k^2\right)$.



Prove that its area is given by $A=\frac{1}{2}k(k-1)$.

4412. A *random walk* on a 3D grid involves a sequence of steps, starting at the origin, each of which, with equal probability, is a move of either $\pm\mathbf{i}$, $\pm\mathbf{j}$ or $\pm\mathbf{k}$. Six steps are made.

Determine the probability of ending up at $(1,1,1)$.

4413. A particle is set in motion at time $t = 0$, and the velocity is modelled as $v = \sqrt{2t+9}$, for $t \geq 0$. A short time δt after the particle is set in motion, displacement from its initial position is described by $\mathrm{s}(\delta t)$, where s is a function.

(a) By definite integration, determine $\mathrm{s}(\delta t)$.
(b) Neglecting terms in δt^3 and higher, work out a polynomial approximation for $\mathrm{s}(\delta t)$.
(c) Show that this approximation is equivalent to the assumption of constant acceleration.

4414. A graph, whose equation is $\mathrm{g}(x) = \mathrm{h}(y)$ for some functions g and h, is rotated 90° clockwise around the origin. Find the equation of the new graph.

4415. By considering a cone as consisting of infinitely many thin discs, use definite integration to prove the formula for the volume:

$$V = \tfrac{1}{3}\pi r^2 h.$$

4416. Sketch the solution set of the inequality

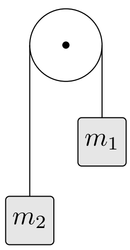
$$y^2 - \sin^2 x \leq 0.$$

4417. Equation E is given as

$$\log_2(x+1) + \log_4 x + \log_8(x+1) = 0.$$

(a) Show that E implies $x^3(x+1)^8 - 1 = 0$.
(b) Show that E has a real root.

4418. Two blocks, $m_1 > m_2$ kg, are connected by a light, inextensible cord which passes over a rough pulley. Friction causes the tensions on either side of the pulley to differ by a maximum of μR Newtons, where R is the downwards force on the pulley and $0 < \mu < 1$ is the coefficient of friction.



The system is in limiting equilibrium. Show that

$$\frac{m_1}{m_2} = \frac{1+\mu}{1-\mu}.$$

4419. A function f is defined over $\mathbb{R}$. For some constant $k \in \mathbb{R}$, f has instruction

$$\mathrm{f}(x) = (1-k)\sin x + k\sin\left(\tfrac{\pi}{2} - x\right).$$

The range of f is $[-A, A]$. Show that

$$A = \sqrt{2k^2 - 2k + 1}.$$

4420. Curve C is defined by $x^2y + 2x + y = 0$.

(a) Show that C has stationary points $(\pm 1, \mp 1)$.
(b) Show that the x axis is an asymptote of C.
(c) Sketch the curve.

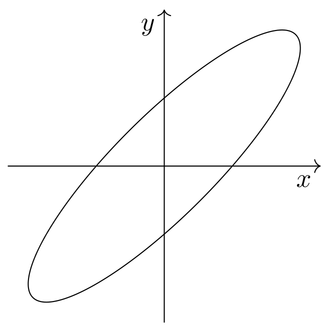
4421. Ladder AB, of length 20 feet, has one end A on flat horizontal ground, and the other end leaning against a vertical wall. End A is slipping away from the wall at a constant speed of 0.3 feet per second. End B remains in contact with the wall.

Show that, when end B is 12 feet above the ground, it is travelling at 0.4 feet per second.

4422. Determine the exact value of the integral

$$\int_1^2 \frac{1}{16t - t^3}\,dt.$$

4423. An ellipse is defined by the parametric equations $x = 2\sin t$, $y = \sqrt{3}\sin t + \cos t$, where $t \in [0, 2\pi)$.



(a) Verify that the Cartesian equation is

$$\left(2y - \sqrt{3}x\right)^2 = 4 - x^2.$$

(b) Show that the curve is tangent to all four of the lines $x = \pm 2$, $y = \pm 2$.

4424. The shape of the normal distribution $X \sim N(0,1)$ is described by the *probability density function*

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}.$$

(a) Show that $\phi(z)$ is stationary at $z = 0$.
(b) Show that $\phi''(\pm 1) = 0$.
(c) Prove that $z = \pm 1$ are points of inflection.

4425. The equations of two circles are given, for some constant $k \in \mathbb{R}$, as

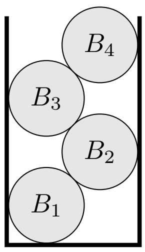
$$x^2 + 6x + y^2 - 8y = 0,$$
$$x^2 - 6x + y^2 + 8y = \frac{200}{k^2+1}.$$

Show that the circles intersect.

4426. Three fair dice are rolled, and the scores are then listed in non-descending order as X, Y, Z. Show that $\mathbb{P}(X + Y = Z) = \frac{5}{24}$.

4427. Show that $\displaystyle\int_0^1 \frac{1}{4x^2+4}\, dx = \frac{\pi}{16}$.

4428. Wine bottles, modelled as cylinders of radius 5 cm and weight 10 N, are stacked in a cellar bin. The bin's width is such that the lines of centres lie at 45° to the horizontal. Friction is assumed to be negligible at all contact points.



(a) Find the width of the interior of the bin.

(b) A set of n bottles is placed in the bin, labelled $B_1, B_2, ..., B_n$. In the example shown, $n = 4$. Find, in terms of n and k, the magnitude of the contact force between bottles B_k and B_{k+1}.

4429. Sketch the curve $x^3 + xy^2 = 1$.

4430. Solve the simultaneous equations

$$(a + 2b)^3 - (a + 2b)^2 = 0,$$
$$2a + 5b = 1.$$

4431. A sculpture consists of an elliptical object, fixed at an angle on a horizontal plinth. Taking $y = 0$ as the level of the plinth, the sculpture is modelled, in units of metres, by the equation

$$2(10x + 5y - 1)^2 + 8(5x - 10y + 2)^2 = 25.$$

(a) Show that $y = 0$ is a tangent to the sculpture.

(b) Verify that the sculpture stands 40 cm above the plinth at its highest point.

(c) Sketch the cross-section.

4432. A student claims that the following equations have the same solution set:

$$|x^2 - x| - 6 = 0,$$
$$|x| \times |x - 1| - 6 = 0.$$

State, with a reason, whether this is true.

4433. Let f be a polynomial function. It is given that $y = \mathrm{f}(x)$ has rotational symmetry around the point (a, b), and that $\mathrm{f}(x) > 0$ for $x \in [0, 2a]$. Prove that

$$\int_0^{2a} \mathrm{f}(x)\, dx = 2ab.$$

4434. Find the following integral:

$$\int \sin x \sin 4x \, dx.$$

4435. From the law of indices $(a^p)^q = a^{pq}$, prove the law of logarithms $\log_a x^n = n \log_a x$.

4436. Random variables X_1 and X_2 are independently distributed as $B(n, p)$, where $n \in \mathbb{N}$ and $p \in (0, 1)$.

(a) Write down the distribution of $X_1 + X_2$.

(b) Show that the distribution of $X_1 - X_2$ is not binomial.

4437. Prove the following identity:

$$\tan 3x \equiv \tan x \tan\left(\tfrac{\pi}{3} - x\right) \tan\left(\tfrac{\pi}{3} + x\right).$$

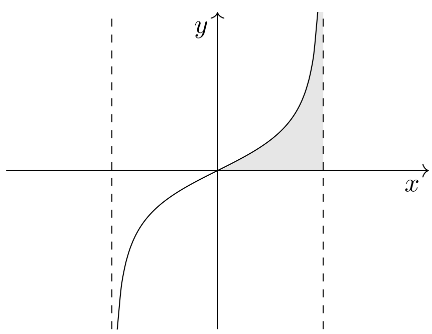
4438. A particle is moving in one dimension according to some function $x = \mathrm{f}(t)$. A simplified quadratic model is then constructed, to describe the motion around time $t = a$. A quadratic model for position $x = \mathrm{g}(t)$ is defined by

$$\mathrm{g}(t) = \tfrac{1}{2}\mathrm{f}''(a)(t - a)^2 + \mathrm{f}'(a)(t - a) + \mathrm{f}(a).$$

Show that, at $t = a$, the two models give matching values for position, velocity and acceleration.

4439. Assuming that θ is a small angle in radians, find a quadratic approximation, in the form $a + b\theta + c\theta^2$, for $(1 - \sin\theta)^{-1}$.

4440. The graph below is $y = \dfrac{x}{\sqrt{1 - x^2}}$



(a) Write down the equations of the asymptotes.

(b) Find $\displaystyle\int \frac{x}{\sqrt{1 - x^2}}\, dx$.

(c) Show that the area of the region between the curve, the x axis, and the positive asymptote is finite.

4441. The first four terms of an increasing sequence are given, for some constants a, b, by

$$a, b, a + b, 2a + b.$$

Show that this sequence is not a GP.

4442. Two functions f and g are defined to be positive, monic quadratics. Each of the equations $\text{f}(x) = 0$ and $\text{g}(x) = 0$ has two distinct, positive, real roots, at $x = a, b$ and $x = b, c$ respectively. The roots $a < b < c$ are in geometric progression. Determine the numbers of roots of the following equations:

(a) $\text{f}(x)\,\text{g}(x) = 0$,

(b) $\text{f}(x) + \text{g}(x) = 0$.

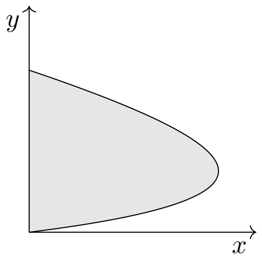
You may wish to use the AM-GM inequality, which states that, for distinct positive $x, y \in \mathbb{R}$,

$$\frac{x+y}{2} > \sqrt{xy}.$$

4443. Show that the circle $x^2 + y^2 = 6$ and the curve $x^2\left(y^2 - x^2\right) = 4$ have eight points of intersection.

4444. A parametric graph is defined by the equations

$$x = 2t - t^2, \qquad y = \tfrac{1}{2}(e^{\frac{1}{2}t} - 1).$$



(a) Show that the parametric integration formula allows for calculation of the area of the shaded region with

$$\int_0^2 (e^{\frac{1}{2}t} - 1)(1 - t)\,dt.$$

(b) Explain, with reference to the sign of $\frac{dx}{dt}$, why this integral produces a negative result.

(c) Show that the exact area is $6 - 2e$.

4445. An artwork shaped as a regular tetrahedron of side length l metres is sitting with one of its faces on flat horizontal ground. Its topmost vertex is at a height of 2 metres. Find l.

4446. A region is enclosed by $y = \pm x$ and the curve

$$\sqrt{x+y} + \sqrt{x-y} = 1.$$

Show that this region has area $\frac{1}{12}$.

4447. Divide $3x^3 - 2x^2y^2 + 3xy - 2y^3$ by $(x^2 + y)$.

4448. For functions f and g, the equation $\text{f}(x) = \text{g}(x)$ has solution set A. A new equation is set up:

$$\text{f}(x)^2 + \text{g}(x)^2 = 2\,\text{f}(x)\,\text{g}(x).$$

Show that this equation has solution set A.

4449. Prove that no tangent to $y = x + \frac{1}{x}$ re-intersects the curve.

4450. In a simulation, n random variables $X_1, X_2, ..., X_n$ are binary, taking values in $\{0, 1\}$. To begin with, they are all set to zero. A random number from the set $\{0, 1, ..., n\}$ is then chosen, which gives the number of X_i variables available for change. Each of these, with probability $1/2$, may then change. Show that the probability that exactly k variables change is given by

$$p = \frac{1}{n+1}\sum_{r=k}^{n} \frac{{}^r\text{C}_k}{2^r}.$$

4451. Heron's formula states that the area of a triangle, side lengths (a, b, c), is given by

$$A^2 = s(s-a)(s-b)(s-c),$$

where s is the semiperimeter $s = \frac{1}{2}(a + b + c)$.

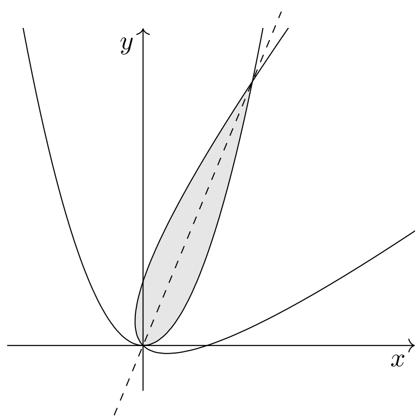
Let T be a triangle with fixed semiperimeter and variable side lengths (a, a, b). Prove that the area of T is maximised if T is equilateral.

4452. A large population is modelled with the normal distribution $X \sim N\big(\mu, \sigma^2\big)$. A sample of size n is to be taken. A slapdash mathematician claims that, for any constant k,

$$\lim_{n\to\infty} \mathbb{P}\left(|\bar{X} - \mu| > k\right) = 0.$$

Correct the claim.

4453. The curve $y = x^2$ is rotated 45° clockwise around the origin.



Show that the parabolae enclose a region of area

$$A = \tfrac{1}{3}\big(7 + 5\sqrt{2}\big).$$

You may use the fact that $\tan 67.5° = 1 + \sqrt{2}$.

4454. Find the area enclosed by the locus of (x, y) points satisfying the relation $|x| + |y| = 1$.

4455. Verify that the parametric equations $x = t - t^3$, $y = 2t + t^3$ may be expressed in Cartesian form as

$$x^3 + 3x^2y + 3xy^2 + y^3 + 18x - 9y = 0.$$

4456. The second-order *Euler-Cauchy equation*, for real coefficients a, b, is defined to be

$$x^2\frac{d^2y}{dx^2} + ax\frac{dy}{dx} + by = 0.$$

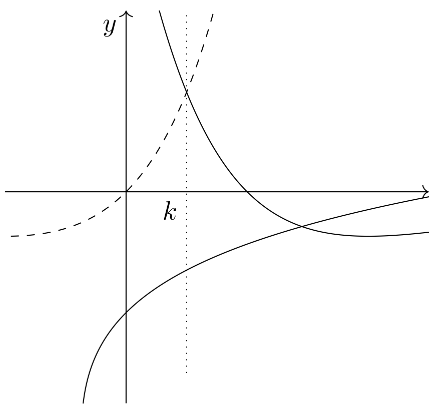
A solution $y = x^k$ is proposed. Show that, if this trial solution is to satisfy the differential equation, then k must satisfy $k^2 + (a-1)k + b = 0$.

4457. Show that, for small x,

$$\frac{x^3 - 7x^2 + 14x - 7}{x^2 - 7x + 12} \approx \frac{119x - 84}{144}.$$

4458. Find $\displaystyle\int \frac{2e^{4x}}{1+e^{2x}}\,dx$.

4459. A graph $y = \text{f}(x)$ (shown dashed below) is reflected in the line $x = k$, then rotated by 90° anticlockwise around the origin.



Show that the image has equation $x = \text{f}(2k + y)$.

4460. A particle moves for $t \in [0, k]$ in one dimension, with velocity given by $v = at^2 + b$, for constants $a, b > 0$. T_0 is defined as the time, in the first k seconds, at which the instantaneous speed is equal to the average speed over the k seconds.

Show that T_0 is independent of a and b.

4461. Solve for the constant k in the equation

$$\int_0^k \sqrt{x}\ln x\,dx = 0.$$

4462. The sum of the first n squares is $\frac{1}{6}n(n+1)(2n+1)$. Using this result, or otherwise, prove that the sum of the first n odd squares is $\frac{1}{3}n(2n+1)(2n-1)$.

4463. Prove, by contradiction, that, if x is a root of the equation $2^{x+1} - 3^{x-1} = 0$, then $x \notin \mathbb{Q}$.

4464. In this question, let n be the number of distinct intersections of the polynomial graphs $y = \text{f}(x)$ and $y = \text{g}(x)$. Find the set of possible values of n when, for $k \in \mathbb{N}$, the functions f and g have degree

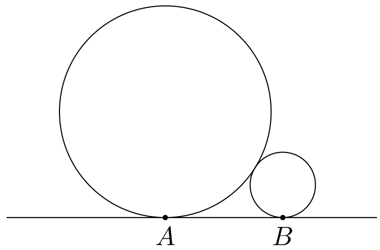
(a) $2k$ and $2k$,

(b) $2k+1$ and $2k+1$,

(c) $2k-1$ and $2k$,

(d) $2k$ and $2k+1$.

4465. Two circles with radii $1/25$ and $1/81$ are placed as shown, tangent to each other and to a line.



Show that $|AB| = \frac{2}{45}$.

4466. Determine the period of the trigonometric function $\text{f}(x) = \sin 4x + \cos 5x + \tan 6x$, defined in radians.

4467. Show that $\displaystyle\frac{d^3}{dx^3}(xy) = 3\frac{d^2y}{dx^2} + x\frac{d^3y}{dx^3}$.

4468. Two sequences are given, for $n \geq 1$, by

$$a_n = 3 \times 2^{n-1},$$
$$b_n = 4 \times 3^{n-2}.$$

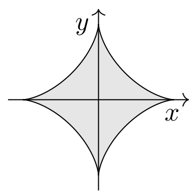
Prove that there is exactly one number which is a term of both sequences.

4469. The function f is defined as $\text{f}(x) = e^{\sin x} + e^{\cos x}$. Show that

(a) $\text{f}(x)$ is locally maximised at $x = \frac{\pi}{4}$,

(b) $\text{f}(x) - 4 = 0$ has infinitely many roots.

4470. An *astroid* is a curve defined, for some constant a, by $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.



(a) Verify that the astroid can be defined in terms of a parameter t by $x = a\cos^3 t$, $y = a\sin^3 t$.

(b) Find the area enclosed by the astroid, giving your answer in the form $k\pi a^2$, where $k \in \mathbb{Q}$.

4471. A quartic curve is defined as $y = 3x^4 - 12x^3 + \varepsilon x$, for some small constant $\varepsilon \in \mathbb{R}$.

(a) Show that, when $\varepsilon = 0$, the quartic curve has a stationary point of inflection at the origin.

(b) Sketch the curve in this instance.

(c) Show that, for small $\varepsilon > 0$, the quartic must have four distinct roots.

4472. The numbers 1 to 16 are placed at random in a 4×4 grid. Find the probability that the squares containing 1 and 2 share an edge.

12	4	11	10
16	15	13	3
1	9	5	7
2	14	6	8

4473. Show that $\displaystyle\int_0^{\frac{\pi}{2}} x^2 \sin x \, dx = \pi - 2$.

4474. A function is called *self-inverse* if $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$.

(a) Show that the function $\mathrm{f} : x \mapsto k - x$ is self-inverse for any constant k.

(b) Prove that no polynomial function of degree 2 or greater is self-inverse.

4475. A scientist is setting up a model for the velocity v of a meteor t seconds after it meets the Earth's atmosphere. The model is

$$v = V + Ae^{-kt}\left(t + \tfrac{1}{k}\right),$$

where A, k, V are constants.

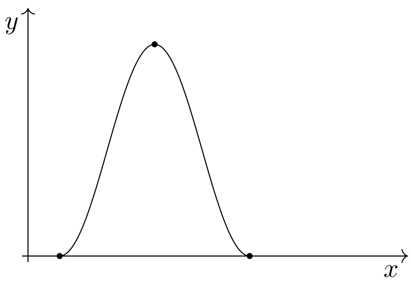
(a) Determine the long-term behaviour, according to the model, and interpret the constant V.

(b) Find the predicted initial acceleration.

(c) Sketch a velocity-time graph.

(d) Find the maximum predicted deceleration.

4476. The diagram below shows, for $x \in [0, 2\pi]$, a part of the graph $\sqrt{y} = \sqrt{3}\sin x - \cos x$.



Find the coordinates of the SPs shown.

4477. State, giving a reason, which of the implications $\implies$, $\impliedby$, $\iff$ links the following statements concerning polynomial functions f and g:

① $\mathrm{f}'(a) = \mathrm{g}'(a)$,

② $\mathrm{f}(x) - \mathrm{g}(x)$ has a factor of $(x-a)^2$.

4478. For any function f, we define functions A and B as

$$\mathrm{A}(x) = \mathrm{f}(x) + \mathrm{f}(-x),$$
$$\mathrm{B}(x) = \mathrm{f}(x) - \mathrm{f}(-x).$$

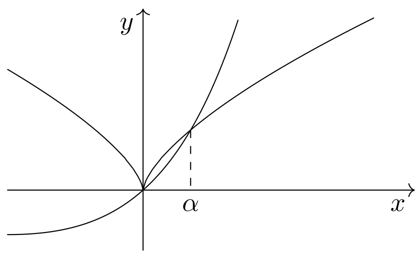
(a) Show that A is even and B is odd.

(b) Prove that any function can be expressed as the sum of an even and an odd function.

4479. Prove that, for small $\theta > 0$ in radians, the secant function may be approximated by a semicircular function:

$$\sec\theta \approx 2 - \sqrt{1 - \theta^2}.$$

4480. The graphs $y = xe^x$ and $x^2 = y^3$ are drawn below.



(a) Show that α is a fixed point of $x_{n+1} = e^{-3x_n}$.

(b) Find α, to 3sf, using a <u>different</u> iteration.

(c) By calculating a suitable derivative, explain why the first iteration diverges very slowly from α.

4481. Determine the value of k for which $y = x\left(e^{2x} + k\right)$ has a stationary point on the x axis.

4482. You are given that the inequalities $\mathrm{g}(x) \geq 0$ and $\mathrm{h}(x) \geq 0$ have solution sets P and Q respectively. For each of the following, state, with a reason, whether the solution set of the given inequality contains the given set.

	Inequality	Set
(a)	$\mathrm{g}(x)\,\mathrm{h}(x) \geq 0$	$P' \cap Q$,
(b)	$\mathrm{g}(x)\,\mathrm{h}(x) < 0$	$P' \cap Q$,
(c)	$\mathrm{g}(x) + \mathrm{h}(x) \geq 0$	$P \cap Q$.

4483. Provide a counterexample to the following claim:

"If the second derivative of a quartic function f has the same value at $x = p$ and $x = q$, then $y = \mathrm{f}(x)$ has a line of symmetry at $x = \frac{p+q}{2}$.

4484. Equations A and B are given as follows:

$$A : xz + yz - x - y = 0,$$
$$B : (z-1)(x+y-1) = 0.$$

Explaining your reasoning carefully, show that, if equations A and B are to hold simultaneously, then $z = 1$.

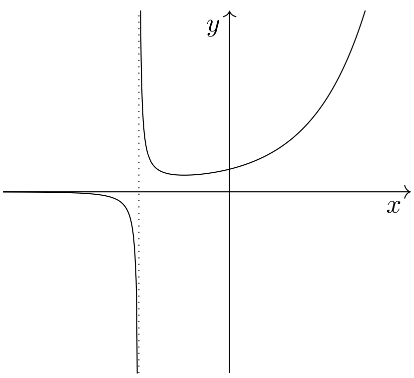
4485. This question is about $\int \log_2 x \, dx$.

(a) Determine $\int \ln x \, dx$.

(b) Hence, show that

$$\int \log_2 x \, dx = x \log_2 x - \frac{x}{\ln 2} + c.$$

4486. The graph of $y = \text{f}(x) = \dfrac{e^x}{x+2}$ is shown below:



Determine the range of f.

4487. One of the following statements, which relate to polynomial functions f and g, is true; the others are not. Prove the one and disprove the others.

(a) If the graph $y = \text{g}(x)$ has an x axis intercept, then so does the graph $y = \text{fg}(x)$,

(b) If the graph $y = \text{g}(x)$ has a stationary point, then so does the graph $y = \text{fg}(x)$,

(c) If the graph $y = \text{g}(x)$ has a point of inflection, then so does the graph $y = \text{fg}(x)$.

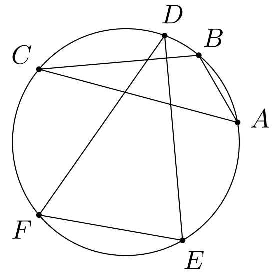
4488. The area of the region bounded by $y = 2xe^{x^2} + 1$, the x axis, $x = 0$, and $x = k$ is e. Determine the value of the constant k.

4489. Show that the locus of points which satisfy the equation $|3x - y| + |x + y| = 2$ is a parallelogram.

4490. Prove a result of Laisant concerning an invertible function f: if $\text{f}(a) = c$ and $\text{f}(b) = d$, then

$$\int_a^b \text{f}(x) \, dx + \int_c^d \text{f}^{-1}(y) \, dy = bd - ac.$$

4491. Six points A, B, C, D, E, F are chosen at random on the circumference of a circle, and triangles ABC and DEF are formed.



Find the probability that the triangles overlap.

4492. Let f and g be functions defined over $\mathbb{R}$. Prove that the second derivative of $\text{f}(x)$ with respect to $\text{g}(x)$ is given by

$$\frac{\text{f}''(x)\,\text{g}'(x) - \text{f}'(x)\,\text{g}''(x)}{(\text{g}'(x))^3}.$$

4493. A pendulum consists of a bob of mass m on the end of a light, inextensible string of length l. Defining θ to be the angle the string makes with the vertical, prove that, for small θ,

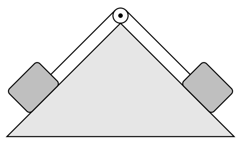
$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta \approx 0.$$

4494. Determine the greatest possible value of

$$y = \sqrt{x^{-\frac{4}{5}} - x^{-\frac{8}{5}}}.$$

4495. Two random variables Y_1 and Y_2 are independent of each other. Each is distributed binomially, as $Y \sim B(5, 1/2)$. Show that $\mathbb{P}(Y_1 > Y_2) = \frac{193}{512}$.

4496. A smooth pulley is rigged up at the top of two 45° slopes, as shown below. The two blocks have masses m and $4m$, and the coefficients of friction between the blocks and the slope are 2μ and μ respectively. The system is in equilibrium.



Find all possible values of μ.

4497. Three lines $y = 0$, $y = mx$, and $y = \sqrt{8}x$ are drawn in the first quadrant. You are given that the line $y = mx$ is the angle bisector of the other two. Determine the value of m.

4498. In a sequences problem, the values of the pth term u_p and the qth term u_q of a GP are given, together with the values of p and q.

With this information, there are values u_n which cannot be calculated with certainty.

Explain the relationship between p and q.

4499. By writing in terms of the variables $s = x + y$ and $t = x - y$, or otherwise, prove that the following equation generates an ellipse:

$$2x^2 - xy + 2y^2 = 1.$$

4500. Describe the quadrilateral $ABCD$ produced by the following construction:

① Place points A at $(0, 0)$ and B at $(4, 0)$.

② Construct circles P and Q with radii 2 and 3, centred at A and B respectively.

③ Translate P by vector $\mathbf{i}$ to form circle P'.

④ Label the intersection of P' and Q point C.

⑤ Translate C by $-\mathbf{i}$. Label this point D.

——— End of 45th Hundred ———